Resonant conductance of two-dimensional electron systems in a long-wave approximation

1991 J. Phys.: Condens. Matter 3 5543

(http://iopscience.iop.org/0953-8984/3/29/008)

J. Phys.: Condens. Matter 3 (1991) 5543–5553. Printed in the UK

Resonant conductance of two-dimensional electron systems in a long-wave approximation

A Matulis and K Patiejūnas
Semiconductor Physics Institute, Lithuanian Academy of Sciences, Požėlos 52, Vilnius, Lithuania, USSR

Received 22 October 1990, in final form 25 February 1991

Abstract. A simple method for the description of resonant phenomena in the ballistic quantum transport of the non-interacting two-dimensional electron gas has been developed. The method is based on the analogies to the theory of diffraction by small holes and is of use when the quantum electronic device consists of some parts separated by apertures the dimensions of which are small compared with the electron wavelength. The conductance of the narrow ring with the magnetic flux through the hole has been calculated and the magnetic field influence on the conductance resonant structure has been considered.

1. Introduction

Recently Wharam *et al* [1] and van Wees *et al* [2] discovered the phenomenon of conductance quantization in the narrow channel of the two-dimensional (2D) electron gas of a GaAs–$Al_xGa_{1-x}As$ heterojunction. Since then a great number of various phenomena related to the interference of electronic waves in two 2D channels, resonant tunnelling and the magnetic field influence on the ballistic quantum transport have been revealed [3–5]. For a theoretical treatment of the ballistic transport in the above-mentioned 2D electronic systems the linear conductance formula $G = (e^2/h)\,\mathrm{Tr}(\mathbf{tt}^+)$, where $\mathbf{t}$ is the transmission amplitude matrix most commonly used. Such an approach can be applied when we are strictly concerned with current sources and voltage measurements at reservoirs connected to the sample [6, 7]. The transmission amplitude matrix calculations are based mainly on the numerical solution of the appropriate Schrödinger equation [8–10]. Those calculations confirmed the quantization phenomena and resonant conductance effects. Nevertheless, we believe the development of the analytical description methods to be of value.

In this paper we present the ballistic transport description method based on the analogies to the theory of diffraction by small holes, developed by Bethe [11]. In our opinion the method could be effectively applied to various resonant phenomena in the ballistic quantum transport.

The paper is arranged as follows. In section 2, Kirchhoff's integral technique for solving the ballistic conductance problem is described and a special integral equation for so-called aperture functions defined in the narrowest 2D device cross sections is derived. In section 3 the above-mentioned integral equation is simplified by means of the long-wave approximation when the electron wavelength is much greater than the

0953-8984/91/295543 + 11 $03.50

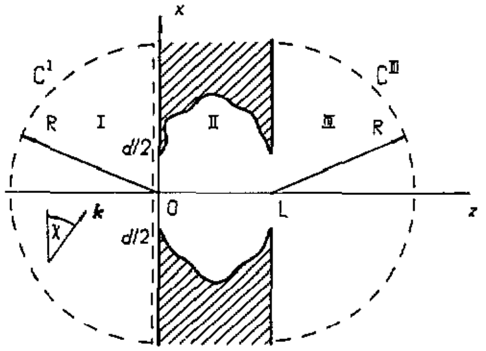

Figure 1. The geometry of the 2D device used for the conductance calculation.

aperture dimensions. The illustration of the suggested method is given in section 4 where the resonant conductance of the 2D ring with the magnetic field is considered. The discussion of the results is presented in section 5.

2. Method of calculation

2.1. Conductance

Let us consider the 2D electronic device with a general layout shown in figure 1. It is made of a central box II connected to two half-planed electrodes I and III by apertures. We assume them to have a size d. Instead of using the transmission amplitude matrix we find it more convenient to use the conductance expression via electron wavefunctions. So, let us introduce the electron eigenfunction Ψ_k which satisfies the Schrödinger equation

$$(H - \tfrac{1}{2}k^2)\Psi_k(x, z) = 0 \tag{2.1}$$

with the hard-wall boundary conditions and the requirement that the asymptotic behaviour in the left-hand half-plane I should correspond to the incident electronic plane wave

$$\Psi_k(x, z) \to \exp_{z\to-\infty}[\mathrm{i}k(x\cos\chi + z\sin\chi)] \tag{2.2}$$

(χ being the angle between the wavevector $\boldsymbol{k}$ and the Ox axis). The Hamilton operator is

$$H = -\tfrac{1}{2}(\partial^2/\partial x^2 + \partial^2/\partial z^2). \tag{2.3}$$

Now, according to [12] we write the following expression for the above-mentioned device conductance:

$$G = J/V = (e^2/\pi\hbar)g \tag{2.4}$$

where V is the voltage applied to the half-planes I and III and J is the current through the central box II. The expression for the dimensionless conductivity g can be written as follows:

$$g = \frac{1}{2\pi}\int_0^{\pi} \mathrm{d}\chi \int_{C^{\mathrm{III}}} \mathrm{d}l\, \mathrm{Im}\left(\Psi_k^* \frac{\partial}{\partial \boldsymbol{n}}\Psi_k\right). \tag{2.5}$$

The first integration in equation (2.5) corresponds to the average over the electrons with the Fermi energy and moving towards the central box, while the second integration gives the total current through the device. We find it convenient to perform this integration along the semicircle C^{III} in the right-hand half-plane. The partial derivative $\partial/\partial \boldsymbol{n}$ has to be calculated in the perpendicular to the contour C^{III} direction.

2.2. *Kirchhoff's integral method*

According to the Green theorem, the wavefunction at any point on the left-hand half-plane I is

$$\Psi_k^{\mathrm{I}}(\boldsymbol{r}') = \oint \mathrm{d}l \left[G^{\mathrm{I}}(\boldsymbol{r}|\boldsymbol{r}') \frac{\partial}{\partial \boldsymbol{n}} \Psi_k^{\mathrm{I}}(\boldsymbol{r}) - \Psi_k^{\mathrm{I}}(\boldsymbol{r}) \frac{\partial}{\partial \boldsymbol{n}} G^{\mathrm{I}}(\boldsymbol{r}|\boldsymbol{r}') \right] \tag{2.6}$$

where the integration should be performed along the contour in the left-hand half-plane indicated in figure 1 by a broken curve. The Green function $G^{\mathrm{I}}(\boldsymbol{r}|\boldsymbol{r}')$ satisfies the following equation:

$$(H - \tfrac{1}{2}k^2) G^{\mathrm{I}}(\boldsymbol{r}|\boldsymbol{r}') = \tfrac{1}{2}\delta(\boldsymbol{r} - \boldsymbol{r}'). \tag{2.7}$$

We assume the Green function to consist of diverging waves only and to satisfy the hard-wall boundary conditions on the interface (including aperture). Then the solution of equation (2.7) can be written in the following way:

$$G^{\mathrm{I}}(\boldsymbol{r}|\boldsymbol{r}') = (\mathrm{i}/4)\left[H_0^{(1)}\left(k\sqrt{(x - x')^2 + (z - z')^2}\right) - H_0^{(1)}\left(k\sqrt{(x - x')^2 + (z + z')^2}\right)\right]. \tag{2.8}$$

Here $H_n^{(1)}$ is the Hankel function of kind one.

Taking into account the Green function boundary conditions and the asymptotic behaviour of function (2.8) on the semicircle C^{I} (when $R \to \infty$), equation (2.6) can be rewritten as

$$\Psi_k^{\mathrm{I}}(\boldsymbol{r}') = \exp(\mathrm{i}k_x x')[\exp(\mathrm{i}k_z z') - \exp(-\mathrm{i}k_z z')] - \int_{-d/2}^{d/2} \mathrm{d}x\, \Phi^{\mathrm{L}}(x) \frac{\partial}{\partial z} G^{\mathrm{I}}(\boldsymbol{r}|\boldsymbol{r}')\big|_{z=0}. \tag{2.9}$$

Here we have introduced the left-hand aperture function

$$\Phi^{\mathrm{L}}(x) = \Psi_k^{\mathrm{I}}(x, 0) \qquad (-d/2 \leqslant x \leqslant d/2). \tag{2.10}$$

By means of the Green theorem, similar expressions for wavefunctions in the central box II and on the right-hand half-plane III can be obtained. They are

$$\Psi_k^{\mathrm{II}}(\boldsymbol{r}') = \int_{-d/2}^{d/2} \mathrm{d}x\, \Phi^{\mathrm{L}}(x) \frac{\partial}{\partial z} G^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}')\big|_{z=0} - \int_{-d/2}^{d/2} \mathrm{d}x\, \Phi^{\mathrm{R}}(x) \frac{\partial}{\partial z} G^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}')\big|_{z=L} \tag{2.11}$$

$$\Psi_k^{\rm III}(r') = \int_{-d/2}^{d/2} {\rm d}x\, \Phi^{\rm R}(x) \frac{\partial}{\partial z} G^{\rm III}(r|r')|_{z=L} \tag{2.12}$$

where

$$\Phi^{\rm R}(x) = \Psi_k^{\rm III}(x, L) \tag{2.13}$$

is the right-hand aperture function. In the diffraction theory, expressions analogous to (2.9), (2.11) and (2.12) are known as Kirchhoff's integrals.

2.3. Equations for aperture functions

The wavefunctions defined by equations (2.9), (2.11) and (2.12) naturally have to satisfy the boundary conditions in the apertures. The equality of the above-mentioned functions in the apertures follows from the Green function definition. The equality of their normal derivatives leads to the following integral equations:

$$\int_{-d/2}^{d/2} {\rm d}x\, K^{\rm L,L}(x'|x)\Phi^{\rm L}(x) + \int_{-d/2}^{d/2} {\rm d}x\, K^{\rm L,R}(x'|x)\Phi^{\rm R}(x) = k_z \exp({\rm i}k_x x') \tag{2.14}$$

$$\int_{-d/2}^{d/2} {\rm d}x\, K^{\rm R,L}(x'|x)\Phi^{\rm L}(x) + \int_{-d/2}^{d/2} {\rm d}x\, K^{\rm R,R}(x'|x)\Phi^{\rm R}(x) = 0 \tag{2.15}$$

the kernels of which can be represented in the form

$$K^{\rm L,L}(x'|x) = -({\rm i}/2) \lim_{\alpha\to 0}\{[\partial^2/(\partial z\, \partial z')][G^{\rm II}(r|r')|_{z'=+\alpha} + G^{\rm I}(r|r')|_{z'=-\alpha}]|_{z=0}\} \tag{2.16}$$

$$K^{\rm R,R}(x'|x) = -({\rm i}/2) \lim_{\alpha\to 0}\{[\partial^2/(\partial z\, \partial z')][G^{\rm III}(r|r')|_{z'=L+\alpha} + G^{\rm II}(r|r')|_{z'=L-\alpha}]|_{z=L}\} \tag{2.17}$$

$$K^{\rm L,R}(x'|x) = -({\rm i}/2)[\partial^2/(\partial z\, \partial z')]G^{\rm II}(r|r')|_{z'=0,z=L} \tag{2.18}$$

$$K^{\rm R,L}(x'|x) = -({\rm i}/2)[\partial^2/(\partial z\, \partial z')]G^{\rm II}(r|r')|_{z'=L,z=0}. \tag{2.19}$$

Here a small positive quantity α is included to avoid the Green function singularity exactly in the aperture.

After the system of aperture equations (2.14) and (2.15) has been solved, the conductivity can be found by substituting $\Phi^{\rm R}$ for equation (2.12) and performing direct integration in equation (2.5).

The main advantage of the suggested equation system (2.14) and (2.15) is the restriction of its definition region. Namely, instead of solving the initial Schrödinger equation (2.1) in the all-device plane, one has to solve the above-mentioned integral equations defined in the apertures only. The kernels are expressed via the Green functions which can be defined separately in the isolated parts of the device. A certain disadvantage of this method is the singularity of the kernels. That is why, for the numerical solution of those integral equations, special methods should be used.

3. Long-wave approximation

Now let us consider a simpler case when the electron wavelength considerably exceeds the aperture dimension, i.e. $kd \ll 1$. In this case the kernels of the integral equations can be expanded into the kd-power series and the equations significantly simplified and reduced to the algebraic equations system. The main problem in performing the above-mentioned long-wave approximation is the Green function singularity when $x \to x'$. This singularity should be singled out in a proper way.

In the case of the left-hand half-plane Green function (2.8) a direct transformation leads to the following expression:

$$\lim_{\alpha\to 0}\{[\partial^2/(\partial z\,\partial z')]G^{\mathrm{I}}(\boldsymbol{r}|\boldsymbol{r}')|_{z=0,z'=-\alpha}\} = \mathrm{i}K_0(x-x')R(k|x-x'|) \tag{3.1}$$

with the singular factor

$$K_0(x) = \lim_{\alpha\to 0}\{(1/2\pi\mathrm{i})[1/(x-\mathrm{i}\alpha)^2 + 1/(x+\mathrm{i}\alpha)^2]\} \tag{3.2}$$

and the regular factor

$$R(x) = (\mathrm{i}x/2)H_1^{(1)}(x). \tag{3.3}$$

Below we restrict ourselves to the following expression for this regular factor:

$$R(x) = 1 + \tfrac{1}{4}\mathrm{i}\pi x^2 \tag{3.4}$$

where only the first non-vanishing real and imaginary power series terms are taken into account.

The Green function $G^{\mathrm{III}}(\boldsymbol{r}|\boldsymbol{r}')$ has a similar expression to that of (2.8). Consequently, the same transformation can be performed.

The Green function for the central box $G^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}')$ is more complicated. Nevertheless, its singular part in the long-wave approximation must coincide with equation (3.1) (within the accuracy of equation (3.4)), since in the above-mentioned approximation the electron wavefunction in the aperture is insensitive to the central-box geometry. This can be directly proved by means of the eigenfunction expansion summation for the central-box Green function.

The above-mentioned Green function properties enable us to use the following expressions for diagonal kernels:

$$K^{\mathrm{L,L}}(x'|x) = K_0(x-x')[1+(\mathrm{i}\pi/4)k^2(x-x')^2] - (\mathrm{i}/2)[\partial^2/(\partial z\,\partial z')]\tilde{G}^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}')|_{z=0,z'=0} \tag{3.5}$$

$$K^{\mathrm{R,R}}(x'|x) = K_0(x-x')[1+(\mathrm{i}\pi/4)k^2(x-x')^2] - (\mathrm{i}/2)[\partial^2/(\partial z\,\partial z')]\tilde{G}^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}')|_{z=L,z'=L}. \tag{3.6}$$

The function $\tilde{G}^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}')$ is the Green function for the central box with the excluded singular part. The exclusion can be performed by means of eigenfunction expansions of the Green function and its singular part (3.1).

Also, the right-hand side of equation (2.14) has to be expanded into the kd-power series as well. In our long-wave approximation case it is sufficient to replace this term simply by the term k_z.

Now, before solving equations (2.14) and (2.15), let us note that the function

$$\Phi_0(x) = -\mathrm{i}(k_z d/2)\sqrt{1-(2x/d)^2} \tag{3.7}$$

exactly satisfies the integral equation

$$\int_{-d/2}^{d/2} \mathrm{d}x\, K_0(x-x')\Phi_0(x) = k_z. \tag{3.8}$$

This enables us to look for the solution of equations (2.14) and (2.15) in the following form:

$$\Phi^{\mathrm{L}}(x) = \Phi^{\mathrm{L}}\Phi_0(x) \tag{3.9}$$

$$\Phi^{\mathrm{R}}(x) = \Phi^{\mathrm{R}}\Phi_0(x). \tag{3.10}$$

Inserting these expressions in equations (2.14) and (2.15) and performing the integration with the accuracy not exceeding $(kd)^2$, we obtain final equations for amplitudes Φ^{L} and Φ^{R}:

$$(1 - \mathrm{i}\eta + K^{\mathrm{L,L}})\Phi^{\mathrm{L}} + K^{\mathrm{L,R}}\Phi^{\mathrm{R}} = 1 \tag{3.11}$$

$$K^{\mathrm{R,L}}\Phi^{\mathrm{L}} + (1 - \mathrm{i}\eta + K^{\mathrm{R,R}})\Phi^{\mathrm{R}} = 0 \tag{3.12}$$

where

$$\eta = (\pi/8)(kd/2)^2 \tag{3.13}$$

and

$$K^{\mathrm{A,B}} = (\pi/4)(d/2)^2[\partial^2/(\partial z\,\partial z')]\tilde{G}^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}')|_{x=x'=0,z=z_{\mathrm{A}},z'=z_{\mathrm{B}}}. \tag{3.14}$$

Here z_{A} and z_{B} are equal to 0 and L for the left- and the right-hand apertures, respectively. Estimating the Green function derivative to be $(\partial/\partial z)G \simeq kG$, one can see that equation (3.14) includes the small parameter $(kd)^2$. Hence, in calculating the conductance, only those Green function parts resonating close to electron energy eigenvalues are significant enough to be taken into account. As the singular Green function part (3.1) does not resonate in the above-mentioned region, in the considered long-wave approximation case the function $\tilde{G}^{\mathrm{II}}$ is to be replaced by G^{II} in equation (3.14).

Substituting equation (3.10) for Kirchhoff's integral (2.12) and later for definition (2.5) we write the following final expression for the dimensionless conductance:

$$g = g_0|\Phi^{\mathrm{R}}|^2 \tag{3.15}$$

where

$$g_0 = \eta^2 \tag{3.16}$$

is the single-aperture conductance in the long-wave approximation [12].

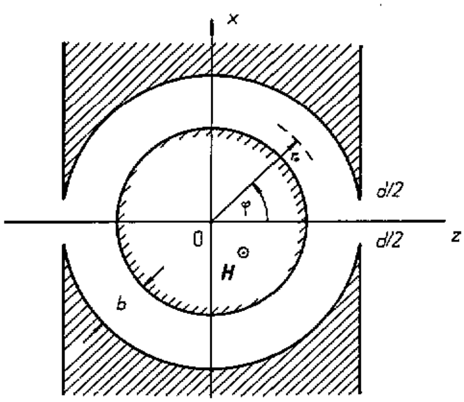

Figure 2. The layout of the 2D ring.

4. 2D ring in magnetic field

To illustrate the suggested method application now let us consider the conductance of the 2D ring, the general layout of which is shown in figure 2. Here r_0 is the radius of the ring and b is its width. Also, we assume there to be a magnetic field with the flux Φ, penetrating through the ring. To describe this magnetic field the vector potential with the axial component

$$A_\varphi = \Phi/2\pi r \tag{4.1}$$

has to be included into the Hamilton operator (2.3). So, to adjust the suggested Kirchhoff's integral method to the above-mentioned problem with the magnetic field, we change the Hamilton operator of the Schrödinger equation (2.1) into

$$H_\Phi = -\tfrac{1}{2}[(1/r)(\partial/\partial r)r(\partial/\partial r) + (1/r^2)(\partial/\partial\varphi + \mathrm{i}\xi)^2] \tag{4.2}$$

where

$$\xi = e\Phi/2\pi\hbar c \tag{4.3}$$

is the dimensionless magnetic flux through the ring. Likewise we change the Hamilton operator of the Green function equation (2.7) into H^*_Φ. This replacement changes the half-plane Green functions $G^{\mathrm{I}}(r|r')$ and $G^{\mathrm{III}}(r|r')$ by the additional phase factor $\exp(\mathrm{i}\xi(\varphi - \varphi'))$. This does not change, however, the main equations (3.11) and (3.12) because within the accuracy of the long-wave approximation this factor can be replaced by unity in the apertures. The definition of the current in the magnetic field has an additional term proportional to the vector potential. Fortunately, it has the zero component perpendicular to the integration contour C^{III} and thus the previous expression for the conductance (2.5) holds.

So, what we have to do is to solve the Green function equation (2.7) with the Hamilton operator H^*_Φ in the central box and to calculate coefficients (3.14) properly.

To simplify the above-mentioned problem, apart from the long-wave approximation $kd \ll 1$, we shall assume the following conditions to be fulfilled: the smallness of the aperture is $d \ll b$ and the narrowness of the ring is $b \ll r_0$. This allows us to use the following approximate Hamilton operator:

$$\bar{H}^*_\Phi = -\tfrac{1}{2}[\partial^2/\partial r^2 + (1/r_0^2)(\partial/\partial\varphi - \mathrm{i}\xi)^2]. \tag{4.4}$$

Now the ring's Green function can be represented as follows:

$$G^{\mathrm{II}}(\boldsymbol{r}|\boldsymbol{r}') = r_0 \sum_{n=1}^{\infty} R_n(r)R_n(r')G_n(\varphi|\varphi') \tag{4.5}$$

where

$$R_n(r) = \sqrt{2/b}\,\sin[(\pi n/b)(r - r_0 + b/2)] \tag{4.6}$$

is the radial eigenfunction and $G_n(\varphi|\varphi')$ is the angular Green function. It obeys the equation

$$[(\mathrm{d}/\mathrm{d}\varphi - \mathrm{i}\xi)^2 + \lambda_n^2(k)]G_n(\varphi|\varphi') = -\delta(\varphi - \varphi') \tag{4.7}$$

where

$$\lambda_n(k) = r_0\sqrt{k^2 - k_n^2} \tag{4.8}$$

and

$$k_n = \pi n/b. \tag{4.9}$$

The solution of equation (4.7) is readily obtainable and the angular Green function can be represented in the following form:

$$G_n(\varphi|\varphi') = -[1/4\lambda_n(k)]\{\exp[\mathrm{i}\lambda_n^+(\varphi - \varphi' \mp \pi)]/\sin(\pi\lambda_n^+) - \exp[\mathrm{i}\lambda_n^-(\varphi - \varphi' \mp \pi)]/\sin(\pi\lambda_n^-)\} \tag{4.10}$$

where

$$\lambda_n^\pm = -\xi \pm \lambda_n(k). \tag{4.11}$$

The upper sign in the right-hand side exponents of equation (4.10) must be used if $\varphi > \varphi'$ and the lower sign when $\varphi < \varphi'$.

Inserting equation (4.10) in (3.14) we obtain the following expressions for the coefficients:

$$K^{\mathrm{L,L}} = K^{\mathrm{R,R}} = \sum_{n=1}^{\infty} A_n(k)\frac{\sin[\pi\lambda_n(k)]\cos[\pi\lambda_n(k)]}{\sin(\pi\lambda_n^+)\sin(\pi\lambda_n^-)} \tag{4.12}$$

$$K^{\mathrm{L,R}} = K^{\mathrm{R,L}} = -\sum_{n=1}^{\infty} A_n(k)\frac{\sin[\pi\lambda_n(k)]\cos(\pi\xi)}{\sin(\pi\lambda_n^+)\sin(\pi\lambda_n^-)} \tag{4.13}$$

where

$$A_n(k) = [\pi r_0/4b\lambda_n(k)](k_n d/2)^2. \tag{4.14}$$

In the long-wave approximation when $|A_n(k)| \ll 1$ it is the resonances of equation

(4.13) that contribute to the ring conductance. To single out these resonances it is convenient to write equations (4.12) and (4.13) as

$$K^{\mathrm{L,L}} = K^{\mathrm{R,R}} = \frac{1}{4br_0} \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} \frac{(k_n d/2)^2}{k^2 - k_{nm}^2} \tag{4.15}$$

$$K^{\mathrm{L,R}} = K^{\mathrm{R,L}} = \frac{1}{4br_0} \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} (-1)^m \frac{(k_n d/2)^2}{k^2 - k_{nm}^2} \tag{4.16}$$

where the ring's eigenvalues are

$$k_{nm} = \sqrt{k_n^2 + (m - \xi)^2/r_0^2} \simeq k_n + (1/2k_n r_0^2)(m - \xi)^2. \tag{4.17}$$

Substituting equations (4.15) and (4.16) into equations (3.11) and (3.12), and solving them in a single-resonance approximation the following expression for the ring conductance is obtained:

$$g = g_0 \sum_{n=1}^{\infty} \sum_{m=-\infty}^{\infty} f_{nm} \frac{\gamma_{nm}/\pi}{(kr_0 - k_{nm} r_0 - \delta_{nm})^2 + \gamma_{nm}^2} \tag{4.18}$$

as a double sum of the Lorentz-type functions shifted with reference to the ring eigenvalues (4.17) by

$$\delta_{nm} \simeq \tfrac{1}{4}(d/2b)(k_n d/2)[1 - (m - \xi)^2/2(k_n r_0)^2] \tag{4.19}$$

with the broadening

$$\gamma_{nm} \simeq (\pi/32)(d/2b)(k_n d/2)^3 \tag{4.20}$$

and the oscillator strength

$$f_{nm} \simeq 1/2\pi n. \tag{4.21}$$

The conductance is seen to consist of the series of spikes corresponding to each radial mode. It is interesting to note that these spikes are more shifted with reference to the ring eigenvalues (4.17) than broadened: $\gamma_{nm}/\delta_{nm} \simeq \eta \ll 1$. The spike position depends on the magnetic flux strength.

This single-resonance approximation is adequate for considering the individual eigenvalue contribution to the ring's conductance. To trace a general view of the motion of the spikes and their interaction in the magnetic field a more sophisticated approximation should be used. Below we write the exact solution for the first radial mode contribution. It is obtained by solving the above-mentioned equations taking into account the first terms in sums (4.12) and (4.13). It reads

$$\begin{aligned} |\Phi^R|^2 = {} & u^2 \sin^2[\pi\lambda_1(k)] \cos^2(\pi\xi) / (\llbracket [(1 - \eta^2)/u]\{\sin^2[\pi\lambda_1(k)] - \sin^2(\pi\xi)\} \\ & - \sin[\pi\lambda_1(k)]\{u \sin[\pi\lambda_1(k)] + 2\cos[\pi\lambda_1(k)]\}\rrbracket^2 \\ & + 4\eta^2 \llbracket (1/u)\{\sin^2[\pi\lambda_1(k)] - \sin^2(\pi\xi)\} - \tfrac{1}{2}\sin[2\pi\lambda_1(k)]\rrbracket^2) \end{aligned} \tag{4.22}$$

where $u = A_1(k)$.

This expression describes the ring conductance when $k \leqslant k_2$ and is depicted as function of $\Delta k\, r_0 = (k - k_1) r_0$ in the range $k_1 \leqslant k \leqslant k_2$ in figure 3. When $k \leqslant k_1$ the conductance has the negligible exponential tail.

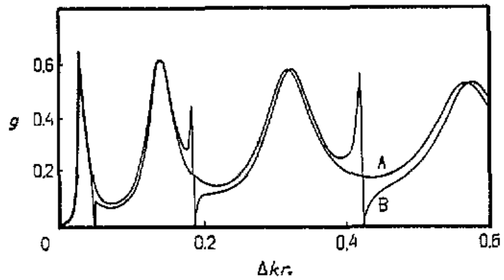

Figure 3. The first radial mode contribution to the ring conductance as a function of $\Delta k r_0 = (k - k_1) r_0$ for $b/r_0 = 0.3$ and $d/b = 0.8$: curve A, $\xi = 0$; curve B, $\xi = 0.1$.

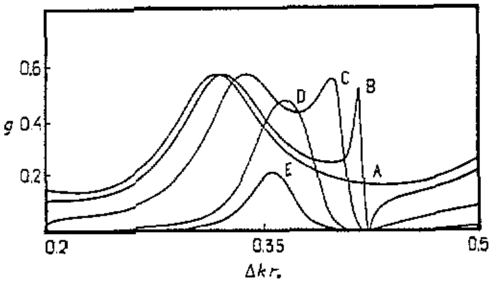

Figure 4. Splitting of the third spike in the magnetic field ($b/r_0 = 0.3$; $d/b = 0.8$): curves A, $\xi = 0$; curve B, $\xi = 0.1$; curve C, $\xi = 0.25$; curve D, $\xi = 0.4$; curve E, $\xi = 0.45$.

5. Discussion of results

The results presented in figure 3 show a well known fact that every series of the ring resonances yields the corresponding series of spikes in the ring conductivity (in figure 3, only one series corresponding to the first radial mode is shown). With the magnetic flux through the ring absent ($\xi = 0$, curve A), each eigenvalue (4.17) is doubly degenerate, which corresponds to two equivalent electron motions in the ring: clockwise and counterclockwise ($\pm m$). When the magnetic field is switched on, these electron motions are no longer equivalent and the degeneracy of the eigenvalues disappears. So, there appear two series of spikes which move with respect to one another as the magnetic field increases (curve B). This situation is shown in figure 4 in some detail where the third spike is depicted. As seen from equation (4.22) the conductance is a periodic function of the magnetic flux magnitude ξ, which is characteristic of Aharonov–Bohm-type phenomena in conducting rings.

It is worthwhile to note two interesting facts related to the magnetic field influence. The first is that the ring conductance vanishes at certain values of the magnetic flux $\xi = l + \frac{1}{2}$, $l = 0, 1, \ldots$ because of the presence of the factor $\cos(\pi\xi)$ in the numerator of equation (4.13). The second is that, when the magnetic field is switched on, the ring conductance vanishes at $k = k_{nm}$ owing to the presence of another factor $\sin[\pi\lambda_n(k)]$ in the same equation (4.13). Thus, as seen from figures 3 and 4, the spike's wing rather than the top is split, which is a direct consequence of the above-mentioned fact that the spikes are more shifted with reference to the ring eigenvalues than broadened. The splitting is very deep and thus for every magnetic flux magnitude one can achieve zero conductivity for a certain value of the Fermi momentum k_{nm} which exactly coincides with ring eigenvalues.

It should be noted that the magntic flux dependence of the ring conductance resonant structure is similar to that obtained in [13] where the Aharonov–Bohm phenomena in a one-dimensional metal ring were studied. They treated the ring connections to current leads phenomenologically, and in the weak-coupling limit, contrary to our results, found that the conductance spikes were more broadened than shifted.

Concluding, we would like to point out that by means of the suggested Kirchhoff's integral method a relatively simple analytical description of resonant features in a ballistic transport can be obtained. We expect this method to be of value for more complicated problems, say those which take elastic electron scattering into account.

References

[1] Wharam D A, Thornton T J, Newbury R, Pepper M, Ahmed H, Frost J E F, Hasko D G, Peacock D C, Ritchie D A and Jones G A C 1988 *J. Phys. C: Solid State Phys.* **21** L209
[2] van Wees B J, van Houten H, Beenakker C W J, Williamson J G, Kouwenhoven L P, van der Marel D and Foxon C T 1988 *Phys. Rev. Lett.* **60** 848
[3] Smith C G, Pepper M, Newbury R, Ahmed H, Hasko D G, Peacock D C, Frost J E F, Ritchie D A, Jones G A C and Hill G 1989 *J. Phys.: Condens. Matter* **1** 6763
[4] van Wees B J, Kouwenhoven L P, Harmans C J P M, Williamson J G, Timmering C E, Broekaart M E I, Foxon C T and Harris J J 1989 *Phys. Rev. Lett.* **62** 2523
[5] Brown R J, Smith C G, Pepper M, Kelly M J, Newbury R, Ahmed H, Hasko D G, Frost J E F, Peacock D C, Ritchie D A and Jones G A C 1989 *J. Phys.: Condens. Matter* **1** 6291
[6] Landauer R 1989 *J. Phys.: Condens. Matter* **1** 8099
[7] Levinson I B 1989 *Zh. Exp. Teor. Fiz.* **95** 2175 (in Russian)
[8] van der Marel D and Haanappel E G 1989 *Phys. Rev.* B **39** 7811
[9] Kirczenow G 1989 *Phys. Rev.* B **39** 10452
[10] Avishai Y, Kaveh M, Shatz S and Band Y B 1989 *J. Phys.: Condens. Matter* **1** 6907
[11] Bethe H A 1944 *Phys. Rev.* **66** 163
[12] Matulis A and Šegžda D 1989 *J. Phys.: Condens. Matter* **1** 2289
[13] Büttiker M, Imry Y and Azbel M Ya 1984 *Phys. Rev.* A **30** 1982